Stability analysis of a hierarchical steering controller with feedback delays for balancing electric scooters in vertical position

Hanna Zsofia Horvath* and Denes Takacs*,†
*Department of Applied Mechanics, Faculty of Mechanical Engineering, Budapest University of Technology and Economics, Budapest, HU
†HUN-REN–BME Dynamics of Machines Research Group, Budapest, HU

Summary. The nonlinear dynamics of electric scooters is investigated using a spatial mechanical model. The nonlinear governing equations are derived with the help of Kane's method. A hierarchical, linear state feedback controller with two feedback delays is designed in order to balance the e-scooter in a vertical position at zero longitudinal speed. The linear stability charts of the delayed controller are constructed with semi-discretization. The optimal higher-level control gains, corresponding to the fastest decay of the solution, are investigated. The performance of the control algorithm is verified by means of numerical simulations.

Introduction

Electric scooters (e-scooters) have recently become very popular in road transportation since they provide great solutions for first and last-mile problems. However, e-scooters of sharing companies often lie everywhere on walkways, causing a high risk of accidents. Thus, in this study, we analyze a possible, futuristic solution by which e-scooters could be driven to designated parking areas. We focus on the stability of a riderless self-driving e-scooter, which can balance itself in the vertical position.

Mechanical model and control design

The balancing task of the e-scooter is analyzed via the spatial mechanical model that is based on the Whipple bicycle model [1]. The multibody system consists of four rigid bodies: the front and the rear wheels, the body (i.e, the frame), and the handlebar and fork assembly, see Fig. 1(a). We derived the nonlinear governing equations of the nonholonomic system with the help of Kane's method [2]. For zero longitudinal speed, the linearized equations of motion can be written as $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{Q}$, where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{Q} = \begin{bmatrix} 0 & M^{\mathrm{s}} \end{bmatrix}^{\mathrm{T}}$ with internal steering torque M^{s}. The vector $\mathbf{x} = \begin{bmatrix} \varphi & \delta \end{bmatrix}^{\mathrm{T}}$ contains the lean angle φ and the steering angle δ. The above-described governing equations agree with the literature [3].

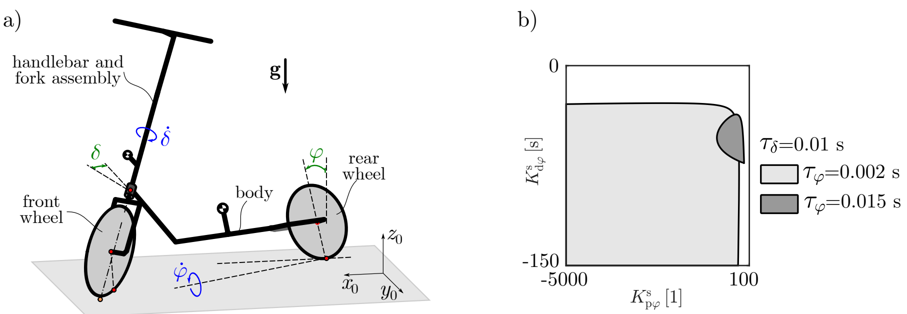

Figure 1: a) The spatial mechanical model of an electric scooter, b) linear stability chart for steering feedback delay $\tau_\delta = 0.01$ s and for two different lean feedback delay values

To balance the e-scooter, we design a hierarchical, linear state feedback controller. A higher-level controller calculates the desired steering angle as

$$\delta_{\mathrm{des}} = -K^{\mathrm{s}}_{\mathrm{p}\varphi}\varphi(t-\tau_\varphi) - K^{\mathrm{s}}_{\mathrm{d}\varphi}\dot{\varphi}(t-\tau_\varphi)\,, \tag{1}$$

where $K^{\mathrm{s}}_{\mathrm{p}\varphi}$ and $K^{\mathrm{s}}_{\mathrm{d}\varphi}$ are proportional gains for the lean angle and the lean rate, respectively. Time delay is taken into account, i.e., the feedback delay τ_φ of the controller is considered in the model. The internal steering torque is created by a lower-level control law as

$$M^{\mathrm{s}} = -K^{\mathrm{s}}_{\mathrm{p}\delta}\left(\delta(t-\tau_\delta) - \delta_{\mathrm{des}}\right) - K^{\mathrm{s}}_{\mathrm{d}\delta}\dot{\delta}(t-\tau_\delta)\,, \tag{2}$$

where $K^{\mathrm{s}}_{\mathrm{p}\delta}$ and $K^{\mathrm{s}}_{\mathrm{d}\delta}$ are proportional gains for the steering angle and the steering rate, respectively. We also consider feedback delay τ_δ of the lower-level controller.

Linear and nonlinear stability analyses

For the delayed controllers, we construct the stability charts with the help of semi-discretization [4]. The stable parameter domains are determined in the plane of the higher-level control gains $K^{\mathrm{s}}_{\mathrm{p}\varphi}$ and $K^{\mathrm{s}}_{\mathrm{d}\varphi}$, see Fig. 1(b). The gray and the white areas correspond to linearly stable and unstable parameter setups, respectively; the different shades of gray relate to different values of the lean feedback delay. The lower-level control gains are fixed, i.e., $K^{\mathrm{s}}_{\mathrm{p}\delta} = 10\,\mathrm{Nm}$ and $K^{\mathrm{s}}_{\mathrm{d}\delta} = -5\,\mathrm{Nms}$ and geometric parameters are based on an existing e-scooter [5].

The effect of the lean feedback delay τ_φ on the linear stability properties can be clearly identified in Fig. 1(b) for fixed steering feedback delay τ_δ. Namely, the greater the lean feedback delay is, the smaller the linearly stable domain is. However, it is worth investigating the optimal higher-level control gains corresponding to the fastest decay of the vibration. Namely, we locate the control gain combinations for which the real part of the rightmost characteristic exponent $\mathrm{Re}\lambda_{\max}$ is the smallest. Figure 2(a) shows the real part of the critical characteristic exponent related to the optimal higher-level control gain setup for different delays. In the upper red part of the colormap, $\mathrm{Re}\lambda_{\max}$ is positive and the motion cannot be stabilized in this large feedback delay domain. However, the best performance of the controller does not correspond to the delay-free case. Let us investigate parameter points A and B of Fig. 2(a). Namely, we fix $\tau_\delta = 0.01\,\mathrm{s}$ and choose $\tau_\varphi = 0.002\,\mathrm{s}$ (point A) and $\tau_\varphi = 0.015\,\mathrm{s}$ (point B). The real part of the rightmost characteristic root is cc. $-5.59\,\mathrm{s}^{-1}$ for point A and cc. $-11.08\,\mathrm{s}^{-1}$ for point B. Therefore, the solution decays faster for a larger lean feedback delay, which is a counterintuitive result.

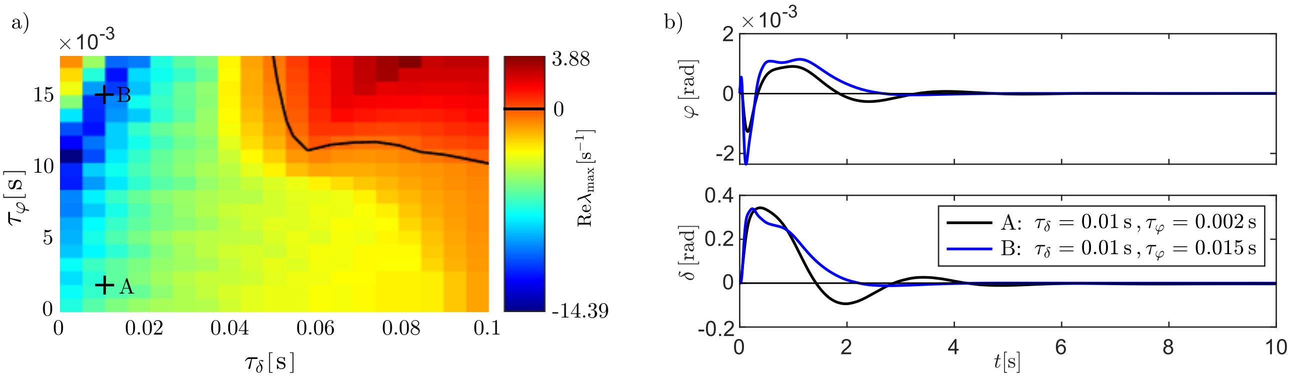

Figure 2: a) The real part of the critical characteristic exponent using the optimal higher-level control gain values of $K^{\mathrm{s}}_{\mathrm{p}\varphi}$ and $K^{\mathrm{s}}_{\mathrm{d}\varphi}$ as a function of the steering feedback delay τ_δ and the lean feedback delay τ_φ, b) nonlinear simulation results using the optimal higher-level control gains of points A and B of Fig. 2(a)

To verify the performance of the controller, numerical simulations are carried out with the nonlinear governing equations and Explicit Euler method with fixed time step and an impact-like perturbation of $\dot{\varphi}(0) = 0.025\,\mathrm{rad/s}$ as an initial condition. Figure 2(b) shows the time graphs of the lean angle φ and the steering angle δ. The black lines refer to point A ($\tau_\delta = 0.01\,\mathrm{s}$, $\tau_\varphi = 0.002\,\mathrm{s}$), with the corresponding higher-level control gains of $K^{\mathrm{s}}_{\mathrm{p}\varphi} = -463.82$ and $K^{\mathrm{s}}_{\mathrm{d}\varphi} = -46.73\,\mathrm{s}$. The vibration decays in time, the lean angle remains small, while the steering angle exceeds 0.35 rad. On the other hand, the vibration decays faster for point B ($\tau_\delta = 0.01\,\mathrm{s}$, $\tau_\varphi = 0.015\,\mathrm{s}$, with the corresponding higher-level control gains of $K^{\mathrm{s}}_{\mathrm{p}\varphi} = -284.42$ and $K^{\mathrm{s}}_{\mathrm{d}\varphi} = -41.46\,\mathrm{s}$), see the blue lines. It can be concluded, that the steering controller designed based on our linear analysis performs well also in the case of large steering angles, even for larger feedback delays.

Conclusions

A hierarchical, linear state feedback controller with two feedback delays was designed to balance an electric scooter in vertical position. It was shown that a larger stable domain may correspond to slower decay of the vibration, and increasing the lean feedback delay can improve the control performance. The experimental validation of the theoretical results is a future task.

Acknowledgements

The research is supported by the National Research, Development and Innovation Office under grant no. NKFI-146201, 2020-1.2.4-TET-IPARI-2021-00012 and partly supported by the Janos Bolyai Research Scholarship of the Hungarian Academy of Sciences. Supported by the UNKP-23-5-BME-299, UNKP-23-3-II-BME-137 New National Excellence Program of the Ministry for Innovation and Technology from the source of the National Research, Development and Innovation Fund. This project has received funding from the HUN-REN Hungarian Research Network.

References

[1] Whipple F. J. W. (1899) The Stability of the Motion of a Bicycle. *Quart. J. Pure Appl. Math.* **30**:312-348.

[2] Kane T. R., Levinson D. A. (1985) Dynamics: Theory and Applications. McGraw-Hill, NY.

[3] Meijaard J. P., Papadopoulos J. M., Ruina A., Schwab A. L. (2007) Linearized Dynamics Equations for the Balance and Steer of a Bicycle: A Benchmark and Review. *Proc. of Royal Society A*, **463**:1955-1982.

[4] Insperger T., Stepan G. (2011) Semi-discretization for Time-Delay Systems. Springer, NY.

[5] Klinger F., Klinger M., Edelmann J., Plochl M. (2022) Electric scooter dynamics – from a vehicle safety perspective. *IAVSD 2021: Advances in Dynamics of Vehicles on Roads and Tracks II*, 1102–1112.